PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Like the crest of a peacock so is mathematics at the head of all knowledge.

1. At a certain time in a deer park, the number of heads and the number of legs of deer and human visitors were counted and it was found there were 39 heads & 132 legs. Find the number of deer and human visitors in the park.
   (Ans:27,12)

Ans: Let the no. of deers be x
And no. of humans be y

ASQ :

$x + y = 39$ ---- (1)
$4x + 2y = 132$ ----- (2)

Multiply (1) and (2)

On solving, we get …

x = 27 and y= 12

∴ No. of deers = 27 and No. of humans = 12

2. Solve for x , y
   a. $\frac{x + y - 8}{2} = \frac{x + 2y - 14}{3} = \frac{3x + y - 12}{11}$ (Ans: x=2, y=6)

Ans: $\frac{x+ y - 8}{2} = \frac{x + 2y - 14}{3} = \frac{3x+ y- 12}{11}$

$\frac{x + y- 8}{2} = \frac{x + 2y - 14}{3}$

On solving, we will get….y= 6

$$\frac{x + y - 8}{2} = \frac{x - 2}{2} = \frac{x + 2y - 14}{3}$$

On solving , we will get….

$x = 2$

b. 7(y + 3) — 2(x + 2) = 14, 4 (y — 2) + 3(x — 3) = 2

Ans: $7(y + 3) - 2(x + 2) = 14$ --------- (1)
$4(y - 2) + 3(x - 3) = 2$ ----------(2)

From (1) $7y + 21 - 2x - 4 = 14$
On solving, we will get….
$2x - 7y - 3 = 0$ ---------- (3)

From (2) $4y - 8 + 3x - 9 = 2$

On solving, we will get….
$3x + 4y - 19 = 0$ ---------- (4)

$2x - 7y - 3$
$3x + 4y - 19$
Substitute this, to get $y = 1$ and $x = 5$

$\therefore \quad x = 5$ and $y = 1$

c. (a+2b)x + (2a— b)y = 2, (a — 2b)x + (2a +b)y = 3 (Ans: $\frac{5b - 2a}{10ab}, \frac{a + 10b}{10ab}$)

Ans:

$2ax + 4ay = y$

, we get $4bx - 2by = -1$

$2ax + 4ay = 5$ $4bx - 2by = -1$
Solve this, to get $y = \frac{10b + a}{10ab}$

Similarly, we can solve for x

d. $\frac{x}{a} + \frac{y}{b} = a + b, \frac{x}{a^2} + \frac{y}{b^2} = 2 ; a \neq 0, b \neq 0$ **(Ans: $x=a^2, y=B^2$)**

Ans: $\frac{x}{a} + \frac{y}{b} = a + b$

$$\frac{x}{a^2} + \frac{y}{b^2} = 2$$

$$\frac{xb + ya}{ab} = a + b$$

$$\frac{xb^2 + ya^2}{a^2b^2} = 2$$

On solving , we get … $x= a^2$ and $y= b^2$

e. $\mathbf{2^x + 3^y = 17, 2^{x+2} - 3^{y+1} = 5}$

Ans: $2^x + 3^y = 17, 2^{x+2} - 3^{y+1} = 5$

Let 2^x be a and 3^y be b

$2^x + 3^y = 17$

$a + b = 17$ ----(1)

$2^{x+2} - 3^{y+1} = 5$

$4a – 3b = 5$ ------(2)

on solving , we get….. $a = 8$

from (1)

$a + b = - 17$

$\therefore b = 9, a = 8$

$\Rightarrow x = 3, y = 2$

f. **If** $\frac{4x-3y}{7x-6y} = \frac{4}{13}$**, Find** $\frac{x}{y}$ Ans: $\frac{4x-3y}{7x-6y} = \frac{4}{13}$

On dividing by *y*, we get $\frac{x}{y} = \frac{5}{8}$

g. **41x + 53y = 135, 53x +41y =147**

Ans: 41x + 53 y = 135, 53 x + 41 y = 147

Add the two equations :

Solve it, to get … x + y = 3 -------(1)

Subtract :

Solve it , to get, …. x – y = 1 --------(2)

From (1) and (2)

x + y = 3
x – y = 1

on solving , we get … x = 2 and y = 1

3. Find the value of p and q for which the system of equations represent coincident lines 2x +3y = 7, (p+q+1)x +(p+2q+2)y = 4(p+q)+1

Ans: $a_1 = 2, b_1 = 3, c_1 = 7$

$a_2 = p + q + 1$, $b_2 = p + 2q + 2$, $c_2 = (p + q) + 1$

For the following system of equation the condition must be

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$=> \frac{2}{p+q+1} = \frac{3}{p+q+2} = \frac{7}{4(p+q)+1}$$

$$=> \frac{2}{p+q+1} = \frac{7}{4(p+q)+1}$$

7p +14q + 14 = 12p + 12q + 3

= 5p - 2q - 11 = 0 ----------(2)

p + q + - 5 = 0
5p - 2q - 11 = 0

From (1) and (2)

5p + 5q - 25 = 0
5p - 2q - 11 = 0

Solve it, to get q = 2
Substitute value of q in equation (1)

p + q - 5 = 0

On solving we get, p = 3 and q = 2

4. Students are made to stand in rows. If one student is extra in a row there would be 2 rows less. If one student is less in a row there would be 3 rows more. Find the number of students in the class.

Ans: No. of rows be y

Let the number of students be x
Number of students in the class will be = xy

One student extra, 2 rows less

(x + 1) (y – 2) = xy

xy – 2x + y -2 = xy
-(-2x + y -2) = 0

One student less, three more rows

(x - 1) (y + 3) = xy

xy + 3x - y -3 = xy
3x - y = 3 ----------(2)

From (1) & (2)

2x – y = -2 X 3
3x - y = 3 X -2

Solve it , to get … y = 12 and x = 5
∴ Number of student = xy
= 12 X 5
= 60 students

5. The larger of two supplementary angles exceeds the smaller by 18^0, find them. (Ans:99^0,81^0)

Ans: $x + y = 180^0$
$x - y = 18^0$

$2x = 198$

$x = 198 / 2 \quad = \quad x = 99^0$

$x + y = 180^0$
$y = 180 - 99$
$y = 81^0$

6. A train covered a certain distance at a uniform speed. If the train would have been 6km/hr faster, it would have taken 4hours less than the scheduled time. And if the train were slower by 6km/hr, it would have taken 6 hours more than the scheduled time. Find the distance of the journey.

Ans: Let the speed of the train by x km/hr
And the time taken by it by y
Now distance traveled by it is x x y = xy

APQ:

I--- $(x + 6)(y - 4) = xy$
$4x - 6y = -24$
$\Rightarrow 2x - 3y = -12$ ----------(1)

II--- $(x - 6)(y + 6) = xy$
$6x - 6y = 36$
$\Rightarrow x - y = 6$ ----------(2)

Solving for x and y we get $y = 24$, $x = 30$

So the distance $= 30 \times 24$
$= 720$ km

7. A chemist has one solution which is 50% acid and a second which is 25% acid. How much of each should be mixed to make 10 litres of 40% acid solution. (Ans:6L,4L)

Ans: Let 50 % acids in the solution be x
Let 25 % of other solution be y

Total Volume in the mixture = x + y

A.P.Q:

$x + y = 10$ --------(1)

A.P.Q: $\frac{50}{100}x + \frac{25}{100}y = \frac{40}{100}\times 10$

$2x + y = 16$ --------(2)

So $x = 6$ & $y = 4$

8. The length of the sides of a triangle are $2x + \frac{y}{2}$, $\frac{5x}{3} + y + \frac{1}{2}$ and $\frac{2}{3}x + 2y + \frac{5}{2}$. If the triangle is equilateral. Find its perimeter.

Ans: $2x + \frac{y}{2}$

$= \frac{4x + y}{2}$ ---------(1)

$= \frac{10x + 6y + 3}{6}$ ---------(2)

$\frac{2}{3}x + 2y + \frac{5}{2}$

$= \frac{4x + 12y + 15}{6}$ ---------(3)

APQ:

$\frac{4x + y}{2} = \frac{10x + 6y + 3}{6} = \frac{4x + 12y + 15}{6}$

$24x + 6y = 20x + 12y + 6$

$2x - 3y = 3$ ---------(4)

$\frac{4x + y}{2} = \frac{4x + 12y + 15}{6}$

$24x + 6y = 8x + 24y + 30$

Solve it,

To get $8x - 9y = 15$ ----------(5)

Solve it ,

To get $x = 3$

Substitute value of x in (4)

$2x - 3y = 3$

Solve it ,

To get $y = 1$

So the values of $x = 3$ and $y = 1$

$$2x + \frac{y}{2} = 6.5 \text{ cm}$$

Perimeter = 6.5 cm + 6.5 cm + 6.5 cm
Perimeter = 19. 5cm

$\therefore$ the perimeter of the triangle is 19.5 cm

8. In an election contested between A and B, A obtained votes equal to twice the no. of persons on the electoral roll who did not cast their votes & this later number was equal to twice his majority over B. If there were 18000 persons on the electoral roll. How many voted for B.

Ans: Let x and y be the no. of votes for A & B respectively.

The no. of persons who did not vote = (18000 – x – y)

APQ:

$x = 2(18000 – x – y)$
$\Rightarrow 3x + 2y = 36000$ ----------(1)

&

$(18000 – x – y) = (2)(x – y)$

$\Rightarrow 3x – y = 18000$ ----------(2)

On solving we get, $y = 6000$ and $x = 8000$

Vote for B = 6000

9. When 6 boys were admitted & 6 girls left the percentage of boys increased from 60% to 75%. Find the original no. of boys and girls in the class.

Ans: Let the no. of Boys be x
Girls be y

Total = x + y

APQ:

$$\frac{x}{x+y} = \frac{60}{100} \text{ ----------(1)}$$

$$\frac{x+6}{(x+6)(y-6)} = \frac{75}{100}$$

On solving we get,

x = 24 and y = 16.

10. When the son will be as old as the father today their ages will add up to 126 years. When the father was old as the son is today, their ages add upto 38 years. Find their present ages.

Ans: let the son's present age be x

Father's age be y

Difference in age (y – x)

Of this difference is added to the present age of son, then son will be as old as the father now and at that time, the father's age will be [y + (y – x)]

APQ:

[x + (y – x)] + [y (y – x)] = 126

[y + (x – y)] + [x + (x – y)] = 38

Solving we get the value of x and y

11. A cyclist, after riding a certain distance, stopped for half an hour to repair his bicycle, after which he completes the whole journey of 30km at half speed in 5 hours. If the breakdown had occurred 10km farther off, he would have done the whole journey in 4 hours. Find where the breakdown occurred and his original speed.
(Ans: 10km, 10km/hr)

Ans*:* Let x be the place where breakdown occurred

y be the original speed

$$\frac{x}{y} + \frac{30-x}{\frac{y}{2}} = 5$$

$$\frac{x+10}{y} + \frac{30-(x+10)}{\frac{y}{2}} = 4$$

$$\frac{x}{y} + \frac{60 - 2x}{y} = 5$$

On solving, we get, x = 10 km and y = 10 km/h

12. The population of the village is 5000. If in a year, the number of males were to increase by 5% and that of a female by 3% annually, the population would grow to 5202 at the end of the year. Find the number of males and females in the village.

Let the number of Males be x and females be y

Ans: $x + y = 5000$

$$x + \frac{5}{100}x + y + \frac{3y}{100} = 5202 \quad \ldots 1$$

$\Rightarrow 5x + 3y = 20200 \quad \ldots 2$

On solving 1 & 2 we get $x = 2600$ $y = 2400$

No. of males = 2600

No. of females = 2400